A frustum is a truncated cone or pyramid; the part that is left when a cone or pyramid is cut by a plane parallel to the base with the apical part removed. We concentrate on the right frustum and not slant frustum as in the right-most diagram on the top.

We begin with:

\[
\text{Volume of prism } = \frac{1}{3}Bh, \quad \text{where } B \text{ is the base area and } h \text{ is the height of prism.}
\]

Since the apical part is removed, we have:

\[
\text{Volume of frustum, } V = \frac{1}{3}(B_2h_2 - B_1h_1) \quad \text{.... (1)}
\]

The height of the frustum \( h = h_2 - h_1 \) \quad \text{.... (2)}

We also have:

\[
\frac{B_2}{B_1} = \frac{h_2^2}{h_1^2} \quad \text{.... (3)}
\]

Our aim is to remove the variables \( h_1, h_2 \) using \( (1) - (3) \).

From (2), \( h_2 = h + h_1 \) \quad \text{.... (4)}

(4) \( \downarrow \) (1), \( V = \frac{1}{3}[B_2(h + h_1) - B_1h_1] = \frac{1}{3}[B_2h + (B_2h_1 - B_1h_1)] \)

\[
= \frac{1}{3}[B_2h + (B_2 - B_1)h_1] \quad \text{.... (5)}
\]

From (3), \( h_2^2 = \frac{h_1^2}{B_1} \Rightarrow h_2 = h_1\sqrt{\frac{B_2}{B_1}} \quad \text{.... (6)}
\]

(6) \( \downarrow \) (2), \( h = h_1\sqrt{\frac{B_2}{B_1}} - h_1 = \sqrt{\frac{B_2}{B_1}}h_1 = h_1 \sqrt{\frac{B_2}{B_1}}h \quad \text{.... (7)}
\]
From (8), we can derive the followings:

(a) Volume of circular cone frustum,
\[ V = \frac{1}{3} [B_2 h + (B_2 - B_1)\frac{\sqrt{B_1}}{\sqrt{B_2} - \sqrt{B_1}} h] \]
\[ = \frac{1}{3} [B_2 h + (\sqrt{B_2} + \sqrt{B_1})(\sqrt{B_2} - \sqrt{B_1})\frac{\sqrt{B_1}}{\sqrt{B_2} - \sqrt{B_1}} h] \]
\[ = \frac{1}{3} [B_2 h + \sqrt{B_1}(\sqrt{B_2} + \sqrt{B_1}) h] \]
\[ = \frac{h}{3} \left( B_1 + \sqrt{B_1}B_2 + B_2 \right) \]

(b) Volume of square frustum,
\[ V = \frac{1}{3} [a_1 h + a_1 a_2 + a_2 h] \]
\[ = \frac{h}{3} \left( a_1 + a_1 a_2 + a_2 \right) \]

(c) Volume of equilateral triangular frustum,
\[ V = \frac{h}{16} \left( a_1^2 + a_1 a_2 + a_2^2 \right) \]
\[ = \frac{h}{16} \left( a_1 + a_1 a_2 + a_2 \right) \]

, where \( r_1, r_2 \) are the radii of the top and bottom circular bases.

, where \( a_1, a_2 \) are the sides of the top and bottom square bases.

, where \( a_1, a_2 \) are the sides of the top and bottom triangular bases.