

Factorization using cross multiplication method

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1. One variable

Most readers are familiar with the cross multiplication method shown below:

(a) $10x^2 - 17x - 20$
 $= (2x - 5)(5x + 4)$

2	-5
×	
5	+4

(b) $8x^2 - 30x + 7$
 $= (4x - 1)(2x - 7)$

4	-1
×	
2	-7

2. Two variables

The method can be extended to two variables. Compare the following examples with those in one variable:

(a) $10x^2 - 17xy - 20y^2$
 $= (2x - 5y)(5x + 4y)$

2	-5
×	
5	+4

(b) $8x^2 - 30xy + 7y^2$
 $= (4x - y)(2x - 7y)$

4	-1
×	
2	-7

3. Biquadratics

The same principle applies to biquadratics:

(a) $10x^4 - 17x^2 - 20$
 $= (2x^2 - 5)(5x^2 + 4)$

2	-5
×	
5	+4

(b) $8x^4 - 30x^2 + 7$
 $= (4x^2 - 1)(2x^2 - 7)$
 $= (2x + 1)(2x - 1)(2x^2 - 7)$

4	-1
×	
2	-7

4. Two variables with constant terms in factors

Factorize : $E = 2x^2 + 2y^2 - 5xy + 7x - 5y + 3$

(i) Form a quadratic function in x :
 $E = 2x^2 - (5y - 7)x + (2y^2 - 5y + 3)$

(ii) Factorize the last term:
 $E = 2x^2 - (5y - 7)x + (2y - 3)(y - 1)$

2	-3
×	
1	-1

(iii) Use cross multiplication again, but you have to deal with variable.

You have to take care of the coefficient of x -term in E :

$$1 \times [-(y - 1)] + 2 \times [-(2y - 3)] = -(5y - 7)$$

$$\begin{aligned} \therefore E &= [x - (2y - 3)][2x - (y - 1)] \\ &= (x - 2y + 3)(2x - y + 1) \end{aligned}$$

1	-(2y - 3)
×	
2	-(y - 1)

5. Three variables

With more twists, the cross multiplication method can be applied to three variables.

(a) Factorize : $E = 2x^2 + 2y^2 + 15z^2 + 5xy - 13yz - 11zx$

(i) Form a quadratic function in x :

$$E = 2x^2 + (5y - 11z)x + (2y^2 - 13yz + 15z^2)$$

(ii) Factorize the last term:

$$E = 2x^2 + (5y - 11z)x + (2y - 3z)(y - 5z)$$

2	-3
×	
1	-5

(iii) Use cross multiplication again, but you have to deal with variables y and z .

You have to take care of the coefficient of the x -term in E :

$$\begin{aligned} 1(y - 5z) + 2(2y - 3z) &= (5y - 11z) \\ \therefore E &= [x + (2y - 3z)] [2x + (y - 5z)] \\ &= (x + 2y - 3z)(2x + y - 5z) \end{aligned}$$

1	+ (2y - 3z)
×	
2	+ (y - 5z)

(b) Factorize : $E = 2x^2 + 2y^2 + 12z^2 - 5xy + 10yz - 11zx$

(i) Form a quadratic function in x :

$$\begin{aligned} E &= 2x^2 - (5y - 11z)x + (2y^2 + 10yz + 12z^2) \\ &= 2x^2 - (5y - 11z)x + 2(y^2 + 5yz + 6z^2) \end{aligned}$$

(ii) Factorize the last term:

$$E = 2x^2 - (5y - 11z)x + 2(y + 2z)(y + 3z)$$

1	+ 2
×	
1	+ 3

(iii) Use cross multiplication again, but you have to deal with variables y and z .

There are several possible cross multiplications. The correct one is shown below.

You have to take care of the coefficient of the x -term in E :

$$\begin{aligned} 1(y + 3z) - 2 \times 2(2y - 3z) &= -(5y - 11z) \\ \therefore E &= [x - 2(y + 2z)] [2x - (y + 3z)] \\ &= (x - 2y - 4z)(2x - y - 3z) \end{aligned}$$

1	- 2(y + 2z)
×	
2	- (y + 3z)

Exercise Factorize the followings

1. $3x^2 - 7x - 6$

2. $16x^2 - 12x - 10$

3. $36x^4 - 229x^2 + 25$

4. $6x^2 - 3xy + 11x - 4y + 4$

5. $10a^2 + 3b^2 + 17ab - 22a - 7b + 4$

6. $4x^2 + 2xy - 2y^2 + 13xz - 5yz + 3z^2$

7. $3x^2 - 2xy - 20xz + 14yz - 7z^2$

Answers

1. $(3x + 2)(x - 3)$

2. $2(2x + 1)(4x - 5)$

3. $(3x - 1)(3x + 1)(2x - 5)(2x + 5)$

4. $(2x - y + 1)(3x + 4)$

5. $(5a + b - 1)(2a + 3b - 4)$

6. $(x + y + 3z)(4x - 2y + z)$

7. $(3x - 2y + z)(x - 7z)$