

QUEEN'S COLLEGE  
Half-yearly Examination, 2009-2010

**S.1 MATHEMATICS PAPER 1**

**MARKING SCHEME**



***General Marking Instructions***

1. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question.
2. In the marking scheme, marks are classified into the following three categories:

‘M’ marks	awarded for correct methods being used;
‘A’ marks	awarded for the accuracy of the answers;
Marks without ‘M’ or ‘A’	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, ‘A’ marks for the corresponding answers should NOT be awarded (unless otherwise specified).
3. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
  - a. The symbol  $(u - 1)$  should be used to denote 1 mark deducted for *u*. At most deduct **1 mark** for *u* for the whole paper.
  - b. The symbol  $(pp - 1)$  should be used to denote 1 mark deducted for *pp*. At most deduct **3 marks** for *pp* for the whole paper. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.
  - c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.
  - d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.
4. All fractional answers must be simplified.

**SECTION A Short questions. (80 marks)**

Answer ALL questions in this section and write your answers in the spaces provided.

<b>1.</b>	<b>(a)</b>	Put the correct symbol '<' or '>' between the two given numbers. (Steps are not required for this question.)	
	<b>(i)</b>	$-7\frac{3}{4}$ , $-7\frac{7}{12}$	(1 mark)
	<b>(ii)</b>	$-15\frac{7}{15}$ , $-15\frac{9}{17}$	(1 mark)
	<b>(iii)</b>	$-2^3$ , $-4^2$	(1 mark)
	<b>(b)</b>	Arrange $-7\frac{3}{4}$ , $-7\frac{7}{12}$ , $-15\frac{7}{15}$ , $-15\frac{9}{17}$ , $-2^3$ , $-4^2$ in ascending order.	(2 marks)
			<b>(5 marks)</b>

<b>(a)</b>	<b>(i)</b>	$-7\frac{3}{4} < -7\frac{7}{12}$	1A	
	<b>(ii)</b>	$-15\frac{7}{15} > -15\frac{9}{17}$	1A	
	<b>(iii)</b>	$-2^3 > -4^2$	1A	
<b>(b)</b>		$-4^2$ , $-15\frac{9}{17}$ , $-15\frac{7}{15}$ , $-2^3$ , $-7\frac{3}{4}$ , $-7\frac{7}{12}$	2A	2A for all correct, but 1A only for correct descending order

<b>2.</b>	It is given that y is a function of x, and $y = x^2(4 + x) - 12$ . Find the value of y when $x = -6$ .	
		<b>(5 marks)</b>

	$y = (-6)^2[4 + (-6)] - 12$	1A	for correct substitution and insertion of 3 ( )s
	$= 36(4 - 6) - 12$	1A 1M	for $(-6)^2 = 36$ for $-(+) = -$
	$= 36(-2) - 12$	1A	
	$= -72 - 12$		
	$= -84$	1A	

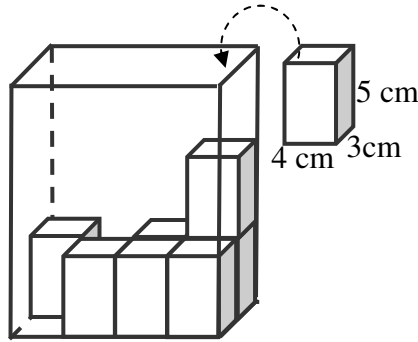
<b>3.</b>	The length and width of a rectangle are 15 cm and $w$ cm respectively. It is known that the perimeter of the rectangle is not greater than 150 cm.		
	<b>(a)</b> Formulate an inequality to represent the above situation.		(2 marks)
	<b>(b)</b> Is it possible that the width of the rectangle is		
	<b>(i)</b> 65 cm, why?		(2 marks)
	<b>(ii)</b> 60 cm, why?		(2 marks)
			<b>(6 marks)</b>

<b>(a)</b>		$2(15 + w) \leq 150$ or $15 + w \leq 75$ or $w \leq 60$	1A 1A	for 'L.H.S.' for ' $\leq$ R.H.S.'
<b>(b)</b>	<b>(i)</b>	No, because	1A	
		$2(15 + 65) = 160 > 150$	1M	
	<b>(ii)</b>	Yes, because	1A	
		$2(15 + 60) = 150$	1M	

<b>4.</b>	Find the value of $P$ when \$ $P$ is deposited in a bank at an interest rate of 12% p.a. for 15 months	
	<b>(a)</b> if the interest obtained is \$300.	(3 marks)
	<b>(b)</b> if the amount obtained is \$32 200.	(4 marks)
		<b>(7 marks)</b>

<b>(a)</b>	$P \times \frac{12}{100} \times \frac{15}{12} = 300$	1M	For using $PRT=I$ and $T=15$ or $\frac{15}{12}$
	$P = 300 \times \frac{100}{15}$	1A	
	$P = 2000$	1A	
<b>(b)</b>	$P \times (1 + \frac{12}{100} \times \frac{15}{12}) = 32\ 200$	1M	For using $A=P(1+RT)$ & $T=15$ or $\frac{15}{12}$
	$P \times (\frac{115}{100}) = 32\ 200$	1A	
	$P = 32200 \times \frac{100}{115}$	1M	For RHS = $32200 \times \frac{100}{115}$ or $\frac{100}{280}$
	$P = 28\ 000$	1A	

5. The figure shows a glass box with 7 rectangular blocks in it. If the length, width and height of the blocks are 4 cm, 3 cm and 5 cm respectively, estimate



<b>(a)</b>	the smallest possible length of the box.	(2 marks)
<b>(b)</b>	the smallest possible width of the box.	(2 marks)
<b>(c)</b>	the smallest possible height of the box.	(2 marks)
<b>(d)</b>	the smallest possible volume of the box.	(2 marks)

<b>(a)</b>	smallest possible length	$\approx / = 4 \times 4$	1A	
		$= 16 \text{ cm}$	1A	
<b>(b)</b>	smallest possible width	$\approx / = 3 \times 2$	1A	
		$= 6 \text{ cm}$	1A	
<b>(c)</b>	smallest possible height	$\approx / = 5 \times 3$	1A	
		$= 15 \text{ cm}$	1A	
<b>(d)</b>	smallest possible volume	$\approx / = 16 \times 6 \times 15$	1M	For using the results in (a), (b) & (c)
		$= 1\,440 \text{ cm}^3$	1A	

<b>6.</b>	<b>(a)</b>	Write an algebraic expression for each of the word phrases below: <b>(i)</b> Subtract $b$ from $a$ . <b>(ii)</b> Divide the difference when $a$ is subtracted from $b$ by the sum of $a$ and $b$ .	(2 marks)
	<b>(b)</b>	If $a = -\frac{1}{3}$ and $b = -\frac{1}{5}$ , find the value of the phrase <b>(a)(ii)</b> .	(6 marks)
			<b>(8 marks)</b>

<b>(a)</b>	<b>(i)</b>	$a - b$	1A	
	<b>(ii)</b>	$\frac{b - a}{a + b}$ or $(b - a) \div (a + b)$	1A	
<b>(b)</b>		$\frac{b - a}{a + b}$ or $(b - a) \div (a + b)$		
		$= \frac{-\frac{1}{5} - (-\frac{1}{3})}{-\frac{1}{3} + (-\frac{1}{5})}$ or $[-\frac{1}{5} - (-\frac{1}{3})] \div [-\frac{1}{3} + (-\frac{1}{5})]$	1M	For correct substitution into <b>(a)(iii)</b> and correct insertion of ( )s
		$= \frac{-\frac{3}{15} + \frac{5}{15}}{-\frac{5}{15} - \frac{3}{15}}$ or $[-\frac{3}{15} + \frac{5}{15}] \div [-\frac{5}{15} - \frac{3}{15}]$	1M	For using 15 as common denominators
		$= \frac{2}{-\frac{8}{15}}$ or $\frac{2}{15} \div (-\frac{8}{15})$	1A	For $\frac{2}{15}$
			1A	For $-\frac{15}{8}$
		$= \frac{2}{15} \times (-\frac{15}{8})$	1M	For rewriting $\div (-\frac{x}{y})$ as $\times (-\frac{y}{x})$
		$= -\frac{1}{4}$	1A	

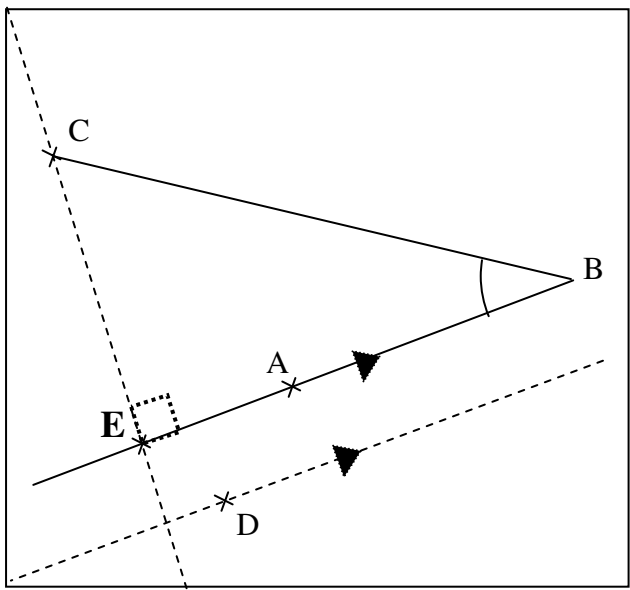
<b>7.</b>	Consider the sequence: $\frac{8}{7}, \frac{9}{14}, \frac{10}{21}, \frac{11}{28}, \dots$	
	<b>(a)</b> Write down the next 2 terms of the sequence.	(2 marks)
	<b>(b) (i)</b> Use an algebraic expression to represent the general term $a_n$ of the sequence.	(2 marks)
	<b>(ii)</b> Use the result of <b>(b)(i)</b> to find the 30 <sup>th</sup> term of the sequence.	(2 marks)
	<b>(iii)</b> If the $m^{\text{th}}$ term of this sequence is $\frac{11}{70}$ , find the value of $m$ .	(3 marks)
		(9 marks)

<b>(a)</b>	the next 2 terms are	$\frac{12}{35}, \frac{13}{42}$	2A	
<b>(b) (i)</b>	the general term	$= \frac{7+n}{7n}$	1A 1A	For numerator For denominator
<b>(ii)</b>	the 30 <sup>th</sup> term	$= \frac{7+(30)}{7(30)}$	1M	For correct substituting $n=30$ into <b>(i)</b> .
		$= \frac{37}{210}$	1A	
<b>(iii)</b>		$\frac{7+m}{7m} = \frac{11}{70}$	1M	For $\frac{11}{70}$ and substituting $m=n$ into <b>(i)</b>
		$490 + 70m = 77m$	1M	For attempt to erase the denominators
		$490 = 7m$		
		$m = 70$	1A	
				0 mark for $m = 4$ or $m = 10$

<b>8.</b>	Solve $-\frac{2}{3}(3x+2) = \frac{x}{2} - \frac{1}{6}(2x-5)$	
		<b>(10marks)</b>

$-\frac{2}{3}(3x+2)$	$= \frac{x}{2} - \frac{1}{6}(2x-5)$		
$-\frac{2}{3} \times 6(3x+2)$	$= 6 \times [\frac{x}{2} - \frac{1}{6}(2x-5)]$ <b>or</b> $= 6 \times \frac{x}{2} - 6 \times \frac{1}{6}(2x-5)$	1M 1A 1M	For multiplying both sides by 6/12/18.... For multiplying both sides by 6 For adding [ ] on R.H.S. or equivalent.
$-4(3x+2)$	$= 6 \times [\frac{x}{2} - \frac{x}{3} + \frac{5}{6}]$ <b>or</b> $= 3x - (2x-5)$	1M 1M	For simplifying $-\frac{2}{3} \times 6(\dots)$ to $-4$ on the L.H.S. For simplifying the R.H.S.
$-12x - 8$	$= 3x - 2x + 5$	1A 1A	For L.H.S. For R.H.S.
$-8 - 5$	$= x + 12x$	1M	For gathering like terms on each side
$13x$	$= -13$	1A	
$x$	$= -1$	1A	
<b>OR</b> $-\frac{2}{3}(3x+2)$	$= \frac{x}{2} - \frac{1}{6}(2x-5)$		
$-2x - \frac{4}{3}$	$= \frac{x}{2} - \frac{x}{3} + \frac{5}{6}$		2A for $-2x - \frac{4}{3}$ , 2A for $-\frac{x}{3} + \frac{5}{6}$
$-2x - \frac{x}{2} + \frac{x}{3}$	$= \frac{4}{3} + \frac{5}{6}$	1M 1A	For gathering like terms on each side
$\frac{-12x}{6} - \frac{3x}{6} + \frac{2x}{6}$	$= \frac{8}{6} + \frac{5}{6}$	1M 1A 1A	For common denominator=6 or 12 For L.H.S. For R.H.S.
$\frac{-13x}{6}$	$= \frac{13}{6}$	1A	
$x$	$= \frac{13}{6} \times \frac{6}{-13}$		
$x$	$= -1$	1A	

**9.** Refer to the given diagram.



- CE (perpendicular line to AB): 1A
- Line thru D parallel to AB: 1A
- Accuracy of the 2 lines (touching the 2 corners): 1A
- Perpendicular symbol: 1A
- Parallel arrows: 1A
- Label E correctly at intersection point of BA and its perpendicular thru C: 1A

**(a)** Name the marked angle in the diagram above in 3 different ways. (1 mark)

$\angle B$ ,  $\angle CBA$ ,  $\angle ABC$

1A for all correct

**(b)** Which type of angle does the marked angle in the diagram belong to? (1 mark)

Acute angle.

1A

**(c)** Arrange the following angles in descending order of size:  
straight angle, obtuse angle, round angle,  
right angle, reflex angle, acute angle. (2 marks)

round angle, reflex angle, straight angle, obtuse angle, right angle, acute angle.

\*2A for all correct

\*Only 1A for correct ascending order.

**(d)** In the above diagram,

**(i)** construct a line passing through D and parallel to AB. (2 marks)

**(ii)** construct a line passing through C and perpendicular AB. (3 marks)

**(iii)** mark the intersecting point of 'the lines in (ii)' and 'the line segment AB' as E. (1 mark)

**(e)** Which 3 points of the above 5 points (A, B, C, D and E) are collinear? (1 mark)

E, A, B

1A for all correct

**(11 marks)**



<b>10.</b>	<b>(a)</b>	Use each of the methods mentioned below to find an estimated value of the expression $225 \times 33$ .	
	<b>(i)</b>	Round <b>off</b> each number in the expression to the nearest ten.	(2 marks)
	<b>(ii)</b>	Round <b>up</b> each number in the expression to the nearest ten.	(2 marks)
	<b>(iii)</b>	Round <b>down</b> each number in the expression to the nearest ten.	(2 marks)
	<b>(b)</b>	In general, which method(s) in <b>(a)</b> may give an estimation smaller than the exact value?	(1 mark)
	<b>(c)</b>	Judy needs to prepare 33 glasses of wine for a party. Each glass can contain 225 mL. Given that the volume of each bottle of wine is 2 L.	
	<b>(i)</b>	Which method in <b>(a)</b> should she use to estimate the number of bottles of wine she needs to buy?	(1 mark)
	<b>(ii)</b>	What is the minimum number of bottles of wine she needs to buy?	(3 marks)
			<b>(11 marks)</b>

<b>(a)</b>	<b>(i)</b>	$225 \times 33$ .		
		$\approx 230 \times 30$ .	1A	
		$= 6900$	1A	
	<b>(ii)</b>	$225 \times 33$ .		
		$\approx 230 \times 40$ .	1A	
		$= 9200$	1A	
	<b>(iii)</b>	$225 \times 33$ .		
		$\approx 220 \times 30$	1A	
		$= 6600$	1A	
			-1 pp	For misuse of ' $\approx$ ' sign(s)
<b>(b)</b>		method (i) & (iii): rounding off and rounding down.	1A	
<b>(c)</b>	<b>(i)</b>	method (ii): rounding up	1A	
	<b>(ii)</b>	$9200 \div 2000$	1M	For using result of <b>(a)(i)</b>
		$= 4.6$	1A	
		The minimum number of bottles of wine she needs to buy is 5.	1A	

**SECTION B Long Questions. (40 marks)**

**Answer ALL questions in this section and write your answers in the spaces provided.**

**Each question carries 20 marks.**

<b>11.</b>	Tom has a certain number of 50 ¢ (\$1 = 100 ¢), \$2 and \$5 coins. The number of 50 ¢ coins is three times that of \$2 coins. The number of \$5 coins is 4 more than twice the number of \$2 coins.			
	<b>(a)</b>	If Tom has twenty-two \$2 coins, find		
		<b>(i)</b>	the number of 50 ¢ coins.	(2 marks)
		<b>(ii)</b>	the number of \$5 coins.	(2 marks)
	<b>(b)</b>	If Tom has $x$ \$2 coins, find		
		<b>(i)</b>	the number of 50 ¢ coins in term of $x$ .	(1 mark)
		<b>(ii)</b>	the number of \$5 coins in term of $x$ .	(1 mark)
	<b>(c)</b>	If the total value of the coins is \$263, find the number of each type of coins.		(14 marks)
				<b>(20 marks)</b>

	<b>(a)</b>	<b>(i)</b>	the number of 50 ¢ coins	= 3 (22)	1A	
				= 66	1A	
		<b>(ii)</b>	the number of \$5 coins	=2(22)+4.	1M	
				= 48	1A	
	<b>(b)</b>	<b>(i)</b>	the number of 50 ¢ coins	= $3x$ ,	1A	
		<b>(ii)</b>	the number of \$5 coins	= $2x+4$ .	1A	

		$2(x) + \boxed{0.5(3x)} + 5(2x + 4) = 263$	2M, 3A	For <u>0.5(bi) or 50(bi)</u> and <u>5(bii)</u> For <u>2(x)</u> , <u>\$0.5(3x)</u> and <u>263</u> in R.H.S.
		$2x + 1.5x + 10x + 20 = 263$	1A	
		$\boxed{13.5}x = \boxed{243}$	1A	
		$x = \frac{\boxed{243}}{\boxed{13.5}}$	1M	For correctly using data from last step.
		$x = (18)$	1A	
		the number of \$2 coins is 18	1A	
		the number of 50 ¢ coins = $3x$		
		= $(18) \times \boxed{3}$	1M	For using result from <b>ai</b> & <b>(8)</b> from last part.
		= 54	1A	
		the number of \$5 Coins is 20 = $2x + 4$		
		= $\frac{\boxed{2(18)+4}}$	1M	For using result from <b>aii</b> & <b>(8)</b> from last part.
		= 40	1A	

<b>12.</b>	A hawker bought 240 eggs for \$ $x$ . Soon, he found 24 of them are rotten. He marked the price of the remaining eggs at \$1.5 each but can only sold half of them in the market. He then sold all the left over eggs out to a shop at 40% discount off the marked price.		
<b>(a)</b>	<b>(i)</b>	Find the number of eggs he sold in the market.	(3 marks)
	<b>(ii)</b>	Find the amount he received from the eggs he sold in the market..	(3 marks)
	<b>(iii)</b>	Find the amount he received from the eggs he sold to the shop.	(4 marks)
	<b>(iv)</b>	Find $x$ if the overall loss per cent of the hawker is 4%.	(6 marks)
<b>(b)</b>	The shop sold all the eggs bought from the hawker out at \$1.35 each. Find the profit per cent of this shop.		(4 marks)
			(20 marks)

<b>(a)</b>	<b>(i)</b>	the number of egg he sold in the market		
		$= \frac{240 - 24}{2}$	1A 1A	for '240 - 24' for $\times 1/2$
		$= \frac{216}{2}$		
		$= 108$	1A	
	<b>(ii)</b>	The total selling price of the eggs he has sold in the market		
		$= \$1.5 \times 108$	1A 1M	For using \$1.5 For using result of (a)
		$= \$ 162$	1A	
	<b>(iii)</b>	The total selling price of the eggs he has sold to the shop		
		$= \$1.5 \times (1 - 40\%) \times 108$	1M 1A	For 108 or (a) For $\$1.5 \times (1 - 40\%)$
		$= \$162 \times (0.6)$ or $= \$0.9 \times 108$	1M	For $\$162 / (b) \times (0.6)$ or $\$0.9 \times 108 / (a)$
		$= \$ 97.2$	1A	

<b>(iv)</b>	$x - 162 - 97.2 = x(0.04)$		1M 1A 1M	for L.H.S. for L.H.S for R.H.S.
	$x - 0.04x = 259.2$		1M	for grouping like terms
	$0.96x = 259.2$		1A	
	$x = \frac{259.2}{0.96}$			
	$x = 270$		1A	
<b>(b)</b>	the profit per cent of this shop.			
	$= 100\% \times \frac{1.35 \times 108 - 97.2}{97.2}$	or	$= 100\% \times \frac{1.35 - 1.5 \times 0.6}{1.5 \times 0.6}$	1M For 97.2 or (c) and 108 or (a)
	$= 100\% \times \frac{145.8 - 97.2}{97.2}$		$= 100\% \times \frac{1.35 - 0.9}{0.9}$	1A
	$= 100\% \times \frac{48.6}{97.2}$		$= 100\% \times \frac{0.45}{0.9}$	1A
	$= 50\%$		$= 50\%$	1A

**END OF PAPER**